The simulation of the stepper motor with variable reluctance by Matlab-Simulink

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Abstract: - This paper is part of a broader study on the behavior of widely used devices in mechatronic systems. In order to harmonize theoretical studies and to support practical research and its application, the balance between the physical component, the math and the optimization - simulation has been taken into account. After having presented the mathematical equations which lead to the stepper motor running's solution, using the Runge-Kutta numerical method in a previous paper, the present one aims to present the Matlab-Simulink model of a simple kinematics, which involves a stepper motor with variable reluctance.

Key-Words: -simulation, stepper motor, variable reluctance, mechatronics, model, kinematics

1.Introduction

Using stepper motors in industrial applications is becoming more apparent. As the diversity of electric motors increases almost daily, manufacturers feel the need to cover market requirements. The developing speed of these quite sophisticated devices always is higher than speed which industrial users can assimilate the improved scientific profile of these devices. For this reason, a lot of industrial companies and even a part of manufacturers require the academic research centers and laboratories to become a turntable between them, simplifying and rushing the assimilation of these high tech devices.

In Romania, the Swiss company BIBUS is one of the biggest distributors and industrial integrators for more than 170 high tech companies worldwide. In order to offer their clients a professional and scientifical support for a huge range of mechatronic devices, including all new generations of motors, BIBUS created in 2008 a *hot scientific point* – "BIBUS TECHNOLOGIC CENTER", inside of the University POLITEHNICA of Bucharest. Here, professors and students make several studies about these complex mechantronic products used by the BIBUS clients and others.

This paper is part of a larger study regarding the integration of stepper motors in certain manufacturing lines and assemblies. The need for precision, stability and rapidity since the stepper motor starts up, imposed various theoretical and experimental studies in order to optimise the input parameters.

1.1. Stepper motors generalities

Stepper motor (SM, see figure 2) is an synchronous brushless electric motor. It is an electromechanical converter that the converts control pulses applied to the motor phases during a rotation. These control pulses consist of discrete angular displacements of equal size, and they represent the steps of the motor. In a proper operation, the number of produced steps must be equal with that of the control pulses applied to the motor phases. [1].

Most of SMs are bidirectional, enabling together acceleration, interruption and fast reversing without steps loss (i.e. higher precision operation), with the only condition of being controlled by a frequency lower than the proper operation regime frequency. To extend the SM operation at speeds higher than the one corresponding to the top frequency, an acceleration by a gradual increasing of the control pulses is needed. SM is used especially in those applications where incremental movement is necessary, applications that use numerical control systems.

The advantages of SMs are the followings:

- can be used in an open circuit;
- can be used in large control frequencies;
- fine precision and resolution (number of steps on

rotation), which simplify the kinematic of the mechanism;
- allows steps lossless starts, stops and reversals;
- can memories the position;
- are compatible with numerical control techniques.

The disadvantages of SMs are the followings:
- fixed value of the step's angle (rotation increment), specified for each SM;
- low yield;
- serious problems if a load with big inertia appears;
- low rotational speed;
- need for a customized and complex control scheme for each SM type, in order to allow a high running speed.

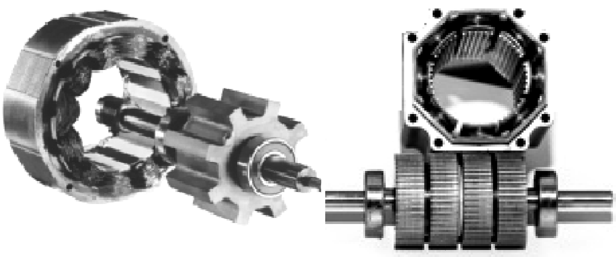
Relatively recent SMs development and the interest in these motors utilization lead to the development of a wide range of MPs range. Basic types of SM are: permanent-magnet Stepper Motors, variable-reluctance Stepper Motors, and hybrid Stepper Motors.

1.2. The variable reluctance of the SMs

Included here, one can find two special designs: SM with a single stator unit (fig. 1, a), and SM with multi stator unit (fig.1, b) [1].

Reluctance is the resistance of a ferromagnetic material to maintain a magnetic field. Reluctance is a parameter similar to the resistance of a conductor when it is crossed by an electric current (it is "*a material's resistance to becoming magnetized*")[2].

Unlike electrical resistance, reluctance is a nonlinear characteristic that depends on the intensity of the magnetic flux. If the reluctance is lower, the magnetic flux passes more easily through the material.



a b

Fig.1 Stepper motors (*a* –single stator unit, *b* – multi stator unit)

Consequently, the materials that can be easily magnetized have a low reluctance, while the nonmagnetic materials have a very high one. The SMs with variable reluctance are motors with passive rotor, with teeth and notches uniformly distributed on the rotor surface. When a stator phase is feeding, the rotor rotates so that the magnetic circuit has minimum magnetic reluctance, meaning that the stator and rotor teeth are radial aligned.

When the power is switched on the next phase, it provides a rotation of the rotor, and the rotor teeth move towards the stator teeth, so that they will not be aligned. This raises a reactive electromagnetic torque which increases the gap between the position angle of stator and rotor teeth. The electromagnetic torque reaches a maximum value, decreasing thereafter to zero. This position is unstable, the rotor teeth being aligned between stator teeth [2].

The stator and rotor of the polistators SMs have the same number of teeth. All statoric packages are mechanically fixed in the same housing, but are independent from electric and magnetic point of view. The rotoric packages are also mechanically fixed on the same shaft and magnetically insulated.

1.3. A common mechanical system that uses a variable reluctance stepper motor

The kinematic scheme (STAS 1543-82) is presented in figure 2.

The system's kinematic is simple, the movement produced by the stepper motor (SM) (ω_1 – angular speed) is transmitted through a rigid coupling (RC) to the driving cogwheel (z_1) which engages with cogwheel z_2 which drives a pinion z_3 (ω_2 - angular speed) of the pinion rack mechanism (R), along the x direction. The rack is mounted on a sled (LS) carrying out high precision linear movement proper to any industrial mechatronic system.

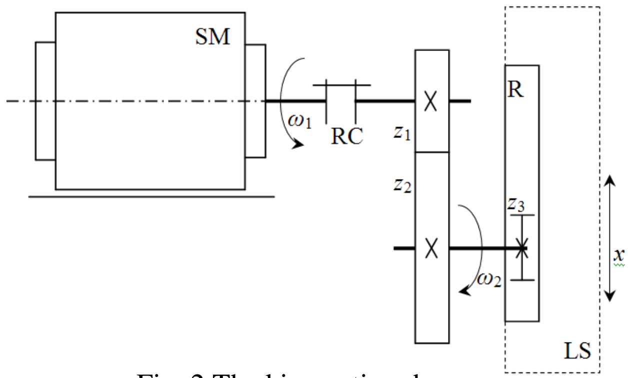


Fig. 2 The kinematic scheme

For the gear z_1/z_2 and the pinion rack mechanism there are defined: m - modulus, z – number of teeth, h – height rack [mm], which is triple than split diameter of the pinion z_3, g – cogwheels width [mm], F_{rez} – resistance force [N], f – frequency [steps/s], p – SM's step angle, m_{LS} – sled weight.

The stepper motor will be chosen in order to accomplish all running conditions imposed by the entire mechatronic system.

As it was already mentioned, the entire theoretical study is larger than this paper topic, and a few theoretical demonstrations were deliberately removed, in order to present just the theoretical aspects related to the Simulink interest [3].

Thus the entire mathematical apparatus for the inertia moments, reduced resistant moment, and inertia moments of the driving system were removed, only the resulted values were used [3].

2. The mathematical model of a stepper motor with two phases, each of them supplied at the rated current

2.1 Stepper motor running equations

The electrical equations of the SM for the two supplied phases are [3]:

$$U_\alpha = R_\alpha \cdot I_\alpha + \frac{d}{dt}\Psi_\alpha = R_\alpha \cdot I_\alpha + \frac{d}{dt}\left(L_{\alpha\alpha} \cdot I_\alpha + L_{\alpha\beta} \cdot I_\beta\right) \quad (1)$$

$$U_\beta = R_\beta \cdot I_\beta + \frac{d}{dt}\Psi_\beta = R_\beta \cdot I_\beta + \frac{d}{dt}\left(L_{\alpha\beta} \cdot I_\alpha + L_{\beta\beta} \cdot I_\beta\right) \quad (2),$$

where:

$$L_{\alpha\alpha} = L_0 + L_p \cdot \cos\left(2p_z\theta_m\right) \quad (3)$$

$$L_{\beta\beta} = L_0 + L_p \cdot \cos\left(2p_z\theta_m - 2p_z \cdot \tau_s\right) \quad (4)$$

$$L_{\alpha\beta} = L_{\beta\alpha} = L_p \cdot \sin\left(2p_z\theta_m\right) \quad (5)$$

The equation of the mechanical running can be written:

$$M_{em} = M_{rr} + J_{red} \cdot \varepsilon_m + D_r \cdot \omega_m, \quad (6)$$

where D_r is viscous friction coefficient.

All terms used in the relations (1)...(6) are: α, β – phases, U – voltage, I – current, R – resistance, ψ – total magnetic flux of the stator coil, L - self inductance, θ_m – angular displacement of the rotor, ω – mechanical angular speed, p – step angle, M_{em} – electro-mechanical moment, Mrr – reduced resistant moment, J_{red} - reduced inertia moment, ω_m – angular speed of the stepper motor.

2.2 The system equations solving by Runge-Kutta numerical method

If the equations system is solved by Runge-Kutta numerical method, it must be converted to one with first degree differential equations.

The electrical equations are mathematically processed and then introduced into a matrix out of which only the first degree derivatives appear:

$$\begin{pmatrix} U_\alpha - R_\alpha \cdot I_\alpha \\ U_\beta - R_\beta \cdot I_\beta \end{pmatrix} = \begin{pmatrix} L_{\alpha\alpha} & L_{\alpha\beta} \\ L_{\alpha\beta} & L_{\beta\beta} \end{pmatrix} \cdot \begin{pmatrix} \frac{dI_\alpha}{dt} \\ \frac{dI_\beta}{dt} \end{pmatrix} + \begin{pmatrix} \frac{\partial L_{\alpha\alpha}}{\partial\theta_m} & \frac{\partial L_{\alpha\beta}}{\partial\theta_m} \\ \frac{\partial L_{\alpha\beta}}{\partial\theta_m} & \frac{\partial L_{\beta\beta}}{\partial\theta_m} \end{pmatrix} \cdot \begin{pmatrix} I_\alpha \\ I_\beta \end{pmatrix} \cdot \frac{d\theta_m}{dt} \quad (7)$$

This matrix system can be arranged in a form that highlights column matrix of each current derivative from both phases:

$$\begin{pmatrix} \frac{dI_\alpha}{dt} \\ \frac{dI_\beta}{dt} \end{pmatrix} = \begin{pmatrix} L_{\alpha\alpha} & L_{\alpha\beta} \\ L_{\alpha\beta} & L_{\beta\beta} \end{pmatrix}^{-1} \cdot \begin{pmatrix} U_\alpha - R_\alpha \cdot I_\alpha \\ U_\beta - R_\beta \cdot I_\beta \end{pmatrix} - \begin{pmatrix} L_{\alpha\alpha} & L_{\alpha\beta} \\ L_{\alpha\beta} & L_{\beta\beta} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \frac{\partial L_{\alpha\alpha}}{\partial\theta_m} & \frac{\partial L_{\alpha\beta}}{\partial\theta_m} \\ \frac{\partial L_{\alpha\beta}}{\partial\theta_m} & \frac{\partial L_{\beta\beta}}{\partial\theta_m} \end{pmatrix} \cdot \begin{pmatrix} I_\alpha \\ I_\beta \end{pmatrix} \cdot \frac{d\theta_m}{dt} \quad (8)$$

where:

$$\frac{\partial L_{\alpha\alpha}}{\partial\theta_m} = -2p_z L_p \sin\left(2p_z\theta_m\right)$$

$$\frac{\partial L_{\beta\beta}}{\partial\theta_m} = -2p_z L_p \sin\left(2p_z\theta_m - 2p_z\tau_s\right)$$

$$\frac{\partial L_{\alpha\beta}}{\partial\theta_m} = 2p_z L_p \cos\left(2p_z\theta_m\right) \quad (9)$$

The differential equation of electromagnetic couple is:

$$M_{em} = \frac{1}{2}\frac{\partial W_m}{\partial\theta_m} = \frac{1}{2}\left(I_\alpha^2 \cdot \frac{\partial L_{\alpha\alpha}}{\partial\theta_m} + I_\beta^2 \cdot \frac{\partial L_{\beta\beta}}{\partial\theta_m}\right) + I_\alpha \cdot I_\beta \cdot \frac{\partial L_{\alpha\beta}}{\partial\theta_m} \quad (10)$$

Also the equation of the mechanical running equation changes to a proper mathematical structure which can be solved by Runge-Kutta numerical method:

$$\frac{d\omega_m}{dt} = \frac{1}{J_{red}} \cdot \left(M_{em} - M_{rr} - D_r \cdot \frac{d\theta_m}{dt}\right) \quad (11)$$

The fourth equation of the system is:

$$\frac{d\theta_m}{dt} = \omega_m \quad (12)$$

This last equation arose from the need to transform the system into a system of first degree differential equations.

The unknown parameters of the system, useful for Matlab-Simulink simulation, are: $I_\alpha, I_\beta, \theta_m, \omega_m$. Initial conditions of the system are: $I_\alpha = 0; I_\beta = 0; \theta_m = 0; \omega_m = 0$, because the SM is in the starting phase. First the motor makes a step and then it stops. Source programs to solve the system of first degree differential equations are made and run in Matlab, and they use a special Matlab function Ode23.

3. The selection of the stepper motor

For choosing the SM, it will be used the numerical modeling described in the previous chapter, as well as in the extended work of the authors [3]. The most important catalog parameters of the steppers are introduced into a few Matlab programs in order to study them. Also, it will record the evolution in time of the current for both phases, of the angular speed, and angular displacement. The variation of these parameters is linked to the mechanical characteristics of the mechanism, such as: reduced inertia moment on motor shaft J_{red}, reduced resistant moment on motor shaft M_{rr}, which directly depends on the resistance force of the LS.

To choose the SM, the specific parameters of the transient regime must be calculated (especially the transient regime of the angular displacement). The stationary value of this regime is 0,0314 rad (1.8°).

For this regime, it will be calculated:
- *Time of first stationary value* (t_{fs}): time variation from the beginning of the transient regime until reaching the first stationary value;
- *Evolution time* (t_e): the measured time until the system raising up to 90% of the stationary value;
- *Initial time* (t_i): the measured time from the beginning of transient regime until the motor reaches the tolerance zone for the first time (tolerance zone is $\pm 3\%$ from stationary value).

After the entire procedure and calculus of choosing stepper motor, for the above mentioned mechatronic devices, it is chosen the stepper motor with best performances from transient regime point of view. The catalog parameters for this motor are:

- step angle = $1.8° = \frac{\pi}{100} rad$;
- step angular precision = 5%;
- nominal current on phase = 8.9 A;
- resistance on phase = 0.3Ω;
- inductance on phase = 2.2 mH;
- motor torque = 1500Ncm;
- moving torque = 70Ncm;
- rotor inertia = 8300gcm^2;
- weight = 10.5kg ;
- t_{fs} = 0,033s; t_e = 0,03s; - t_i = 1,95s.

This motor reaches with a higher precision the stationary value of the angular displacement and this helps to increase the precision of the mechatronic systems [6].

A common disadvantage (not so important) is that the parameters of the transient regime of the angular displacement are negatively influenced by the higher inertia moment of the stepper rotor.

4. System simulation using Matlab-Simulink

The aim of this simulation is to emphasize the first second of stepper running, when the undesirable dynamic phenomena appear [7]. The most important parameters when choosing a SM are:

Nominal current of both phases: $I_\alpha = I_\beta = 8.9A$

Resistance of both phases: $R_\alpha = R_\beta = 0.3\Omega$

Inductance of both phases: $L_0 = 2.2 \cdot 10^{-3} H$

Inertia moment of rotor: $J_{rotor} = 8300 \cdot 10^{-7} Nm^2$

The stepper motor is controlled by several types of signals and the system answers are analyzed. The best control is thus chosen.

For this control, the system response will be tested related to variations in time of the reduced resistance moment. To achieve this, it will use a special model realized in Simulink (fig. 3) and the simulation results will be graphically represented (fig.4…9).

The most known types of control used in simulation are applied to the stepper motor (step, ramp and sinusoidal) [8].

For those familiar with modeling and analysis in environment Matlab-Simulink, the model presented in figure 3 is sufficiently clear in terms of consistency between the used blocks and the scientific fundamentals of this kind of motor. The “hidden” part of this model is the content of the three subsystems from the left side of the model (fig. 3). This paper does not present the content of these subsystems and the correlations and links with the model, only for editorial space reasons.

Step control command is often used in such simulations where the results are more than satisfactory for transient regime parameters. Representation of the parameters of interest are shown in figures 4…7 I_a, I_b, ω_m, θ_m.

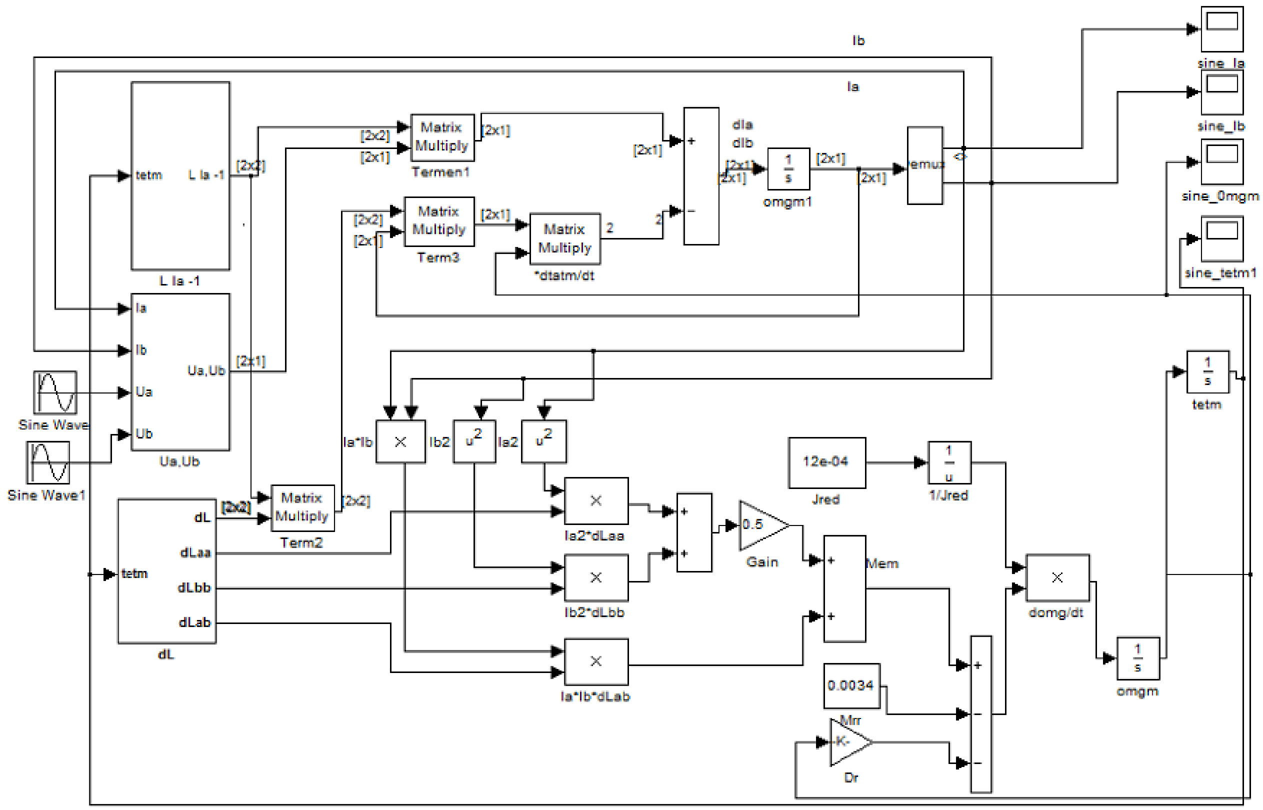


Fig. 3 The Matlab-Simulink model

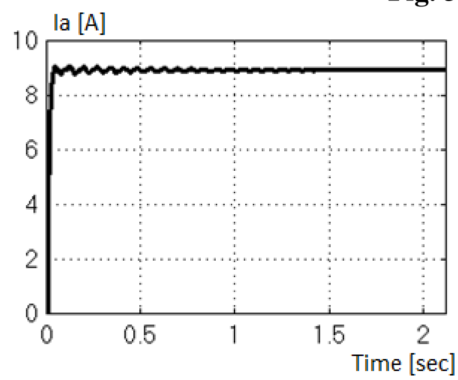


Fig. 4 The *Ia* characteristic (step signal)

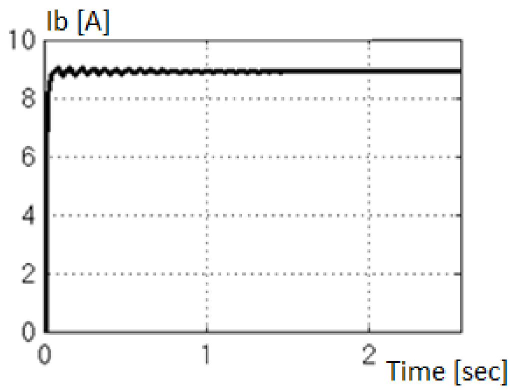


Fig. 5 The *Ib* characteristic (step signal)

It can observed that the signal is quickly stabilized at a proper value of current (8,9 A) for both phases. Also angular speed and angular displacement have a symmetrical variation that means the vibrations are not so dangerous versus

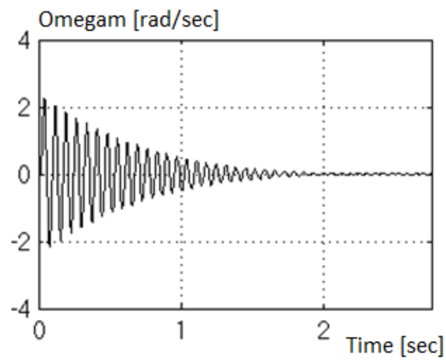


Fig. 6 The ω_m characteristic (step signal)

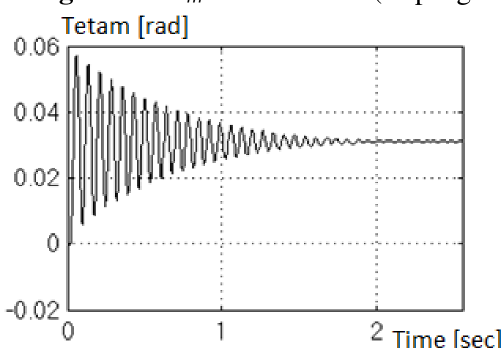


Fig. 7 The θ_m characteristic (step signal)

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the case of other control commands (e.g. of ramp and sine way), when signal appears lop-sided or repeated (fig 8 and fig. 9).

Ramp control command (fig.8) offers results with important errors for the parameters of angular displacement and angular speed, improper for this kind of mechatronic application. When the slope of the ramp signal increases, the period of signal utilisation decreases, and some better dynamic characteristics could be obtained. Even so, they still remain far of the application demands.

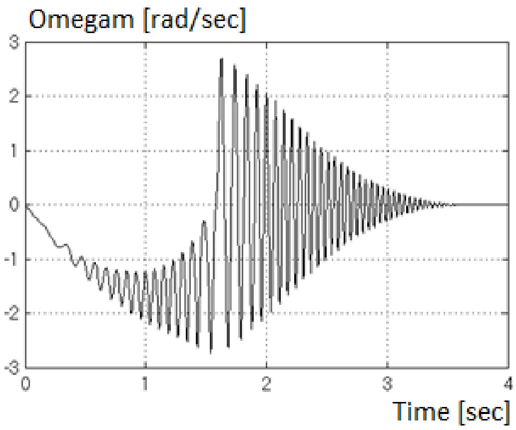

Fig. 8 The ω_m characteristic (ramp signal)

Sine wave control command (fig.9) is almost similar to step control command, but it has the disadvantage of higher value of angular displacement exactly in the area where signal must be stabilized. In this case, the stabilization time increases and it is possible the signal not to be stable any more.

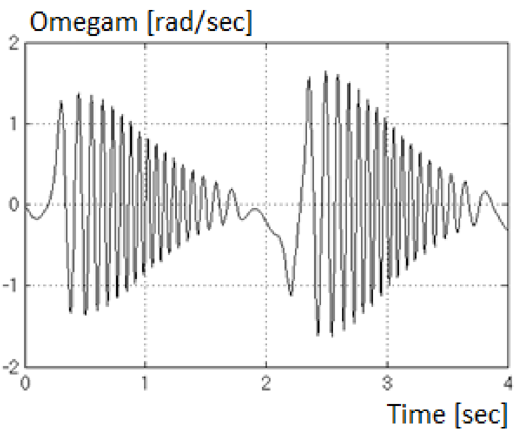

Fig. 9 The ω_m characteristic (sine wave signal)

5. Conclusions

The conclusion reached as a result of this comparative analysis is that the *step* control can be the most useful command for the stepper motor in our application. In angular displacement representation, it can be observed that, after stabilization, some small sinusoidal oscillations appear, following the variation of the reduced resistant moment. But the system response from angular displacement is satisfactory. These low vibrations of the reduced resistant moment have no influence on the rotor angle which has small oscillations around stabilization position. These oscillations are not so important and do not have a real influence on the proper stepper motor running.

Another part of the study presented in another paper, in order to complete the information about this type of stepper motor, is a study of the dynamic stability, which used the transfer function and Bode diagrams of phase and amplitude, also realized in Matlab-Simulink.

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