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Modeling and solving building physics problems with FemLab

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Abstract

The commercially available software package FemLab is evaluated as solver for building physics problems based on partial differential equations (PDEs). The software is designed to simulate systems of coupled PDEs which may be 1D, 2D or 3D, non-linear and time dependent. An important feature of FemLab is that the user can focus on the model (PDE coefficients on the domain and boundary) and does not have to spend much time on solving and visualization. In this paper, 4 cases are considered. First, in order to illustrate how FemLab works, an example including the complete code for solving as well as the results are given for a simple 2D steady-state heat transfer problem. In the next 2 cases, the reliability is tested for two very different building physics problems: A 2D dynamic airflow problem, modeled using Navier–Stokes and buoyancy equations, and a 1D dynamic non-linear moisture transport in a porous material. These simulation results are validated and show a good agreement with measurements. In the last case, FemLab's capability of simulating 3D problems is shown by a dynamic combined heat and moisture transport problem. This example is a 3D extension of a given 2D problem from IEA Annex 24 (Final Report—Task 1). For all models the crucial part of the codes (geometry, PDEs and boundary specifications) are given. The FemLab software is written in the MatLab environment (The Mathworks, Inc. MatLab manual, Version 5.3, 1998) and therefore it is possible to use the visualization tools, toolboxes and all other programs written in MatLab. The evaluation illustrates the powerful and flexible nature of FemLab for solving scientific and engineering building physics problems. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Many scientific problems in building physics can be described by PDEs. There are a lot of software programs available in which one specific PDE is solved. They are developed in order to get the simulation results in a short time and most often a lot of effort has been put into the simplicity of input of data, e.g. geometrical data. A disadvantage is that they often are not very flexible when the user wants to change or combine models. Another drawback is that they most often act as black boxes. Another category of commercially available software like FemLab [1] is developed specifically for solving PDEs where the user in principle can simulate any system of coupled PDEs. Practical physics/engineering problems in the area of heat transfer, electromagnetism, structural mechanics and fluid dynamics can be solved with the software. The practical problems solved in this paper are: a 2D thermal bridge problem, a 1D moisture transport problem, a 2D airflow problem and a 3D combined heat and moisture transport problem. One of the main advantages of FemLab is that the user can focus on the model (PDE coefficients on the domain and the domain boundary) and does not have to spend much time on solving and visualization. The scientist can concentrate on the physics behind the models and the engineer can calculate details for designing purposes using validated models.

Section 2 shows some features of FemLab and how it works by a simple 2D steady state heat transfer problem. In Section 3 the quality of the numerical solvers is tested by solving two very different building physics problems: A 2D dynamic airflow problem and a 1D dynamic non-linear

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Nomenclature

Gr Grasshof number (dimensionless)
bitheat transfer coefficient (W/m² K) $PDE coefficients$ Kheat conductivity (W/m K) $a, c, d_a, f, F, g, G, h, q, r, R, \alpha, \beta, \gamma, \Gamma$ noutward unit normal (dimensionless)
ppressure (Pa)SubscriptsPrPrandl number (dimensionless)Subscripts
ReReynolds number (dimensionless)0initial value at $t = 0$ ttime (s)0initial value at $t = 0$ Ttemperature (°C), ((dimensionless) when scaled)eexternalusolution of the PDE(s), e.g. temperature, mois- ture content, etc.concrconcreteu, vvelocity (scaled) (dimensionless)maxmaximum

moisture transport in a porous material. The solutions are compared with measurements. There are not many software packages available which are capable of simulating 3D dynamic combined heat and moisture transport problems. In Section 4 it is shown how this can be done using FemLab.

2. How FemLab works

FemLab [1] is a toolbox written in MatLab [2]. It solves systems of coupled PDEs (up to 32 independent variables). The specified PDEs may be non-linear and time dependent and act on a 1D, 2D or 3D geometry. The PDEs and boundary values can be represented by two forms. The *coefficient* form is as follows:

$$d_{a} \frac{\partial u}{\partial t} - \nabla \cdot (c \nabla u + \alpha u - \gamma) + \beta \nabla u + a u = f \quad \text{in } \Omega$$
 (1a)

$$\underline{n} \cdot (c\nabla u + \alpha u - \gamma) + qu = g - \lambda \qquad \text{on } \partial\Omega \quad (1b)$$

$$hu = r$$
 on $\partial \Omega$ (1c)

The first equation (1a) is satisfied inside the domain Ω and second (1b) (generalized Neumann boundary) and third (1c) (Dirichlet boundary) equations are both satisfied

on the boundary of the domain $\partial\Omega$. *n* is the outward unit normal and is calculated internally. λ is an unknown vector-valued function called the *Lagrange multiplier*. This multiplier is also calculated internally and will only be used in the case of mixed boundary conditions. The coefficients d_a , c, α , β , γ , a, f, g, q and r are scalars, vectors, matrices or tensors. Their components can be functions of the space, time and the solution u. For a stationary system in coefficient form $d_a = 0$. Often c is called the *diffusion coefficient*, α and β are *convection coefficients*, a is the *absorption coefficient* and γ and f are *source terms*.

The second form of the PDEs and boundary conditions is the *general* form:

$$d_{a} \frac{\partial u}{\partial t} + \nabla \cdot \Gamma = F \quad \text{in } \Omega,$$
 (2a)

$$-\underline{n} \cdot \Gamma = G + \lambda \quad \text{on } \partial\Omega, \tag{2b}$$

$$R = 0 \qquad \text{on } \partial\Omega. \qquad (2c)$$

The coefficients Γ and F can be functions of the space, time, the solution u and its gradient. The components of G and R can be functions of the space, time, and the solution u.



Fig. 1. The geometry of the 2D thermal bridge example.

Table 1

Boundary specifications of the 2D thermal bridge problem, where T is the temperature on the boundary and T_i and T_e , respectively are the internal and external temperatures

Boundary segment	Boundary type	Boundary segment length (m)	Boundary condition (W/m^2)
b1	External	1.0	$\varphi = hc_e^*(T_e - T)$
b2	Adiabatic	0.2	$\varphi = 0$
b3	Adiabatic	0.1	$\varphi = 0$
b4	Internal	0.8	$\varphi = hc_i^*(T_i - T)$
b5	Internal	0.7	$\varphi = hc_i^*(T_i - T)$
b6	Adiabatic	0.2	$\varphi = 0$
b7	Adiabatic	1.0	arphi=0

Given the geometry data, an initial finite element mesh is automatically generated by triangulation of the domain. The mesh is used for discretisation of the PDE problem and can be modified to improve accuracy. The geometry, PDEs and boundary conditions are defined by a set of fields similar to the structure in the language C. A graphical user interface is used to simplify the input of these data. For solving purposes FemLab contains specific solvers (like static, dynamic, linear, non-linear solvers) for specific PDE problems.

Example. FemLab code and results of a 2D stationary thermal bridge.

A 2D stationary thermal bridge problem is used as an example of how FemLab works. In Fig. 1 the geometry of the 2D thermal bridge problem is shown. In Table 1 the lengths and boundary conditions of each boundary segment are given. The PDE model for the inside of the domain is given by

$$\nabla \cdot (K\nabla T) = 0, \tag{3}$$

where *K* is the heat conductivity and *T* is the temperature. Using the coefficient form (1a) and the model (3), it follows that *u* equals *T* and the coefficients of (1a) are all zero $(a = d_a = f = \alpha = \beta = \gamma = 0)$ except *c*. The *c* coefficient equals the heat conductivities at the sub-domains concrete (K_{concr}) and insulation (K_{insul}) . The boundary values are heat fluxes and so the Neumann condition is applied. For example, boundary condition b1: $\varphi = hc_e^*(T_e - T)$ is represented by taking $q = hc_e$, $g = hc_e^*T_e$ in Eq. (1b). Note

*CONSTAN	TS	
hi=7.7;	<pre>%heat transfer coefficient</pre>	internal
he=25;	<pre>%heat transfer coefficient</pre>	external
Ti=20;	%air temperature internal	
Te=-10;	%air temperature external	
Kconcr=1;	Sheat conduction concrete	
Kinsul=0.03;	*heat conduction insulation	'n
%GEOMETR CONCR=poly2([0 INSUL=poly2([0 fem.geom=CONCR fem.dim=1;	Y: poly2(XDATA,YDATA) ; 2D 0 1 1 0.2 0.2],[0 1 1 0.8 .2 0.2 1 1],[0.7 0.8 0.8 0. +INSUL;	polygon 0.8 0]); %concrete 7]); %insulation %fem geometry %One component
%COEFFIC	IENTS OF THE PDE/Boundary p	roblem
fem.equ.c={Kco	ncr Kinsul };	% fem coefficient c
fem.bnd.g={he*	Te 0 0 hi*Ti hi*Ti 0 0};	% fem coefficient g
fem.bnd.q={he	0 0 hi hi 0 0};	% fem coefficient q
fem.mesh=meshi	nit(fem);	% intialize mesh
fem.sol=femlin	(iem);	% solve, steady problem
%OUTPUT meshplot(fem) q=posteval(fem postplot(fem,'	MESH, SOLUTION ,'u'); tridata',q,'tribar','on')	% plot mesh % post processing data % plot solution

Fig. 2. The complete FemLab code for solving the 2D thermal bridge problem.



Fig. 3. The mesh of the 2D thermal bridge problem.



Fig. 4. The solution (temperature distribution) of the 2D thermal bridge problem.

that the term $\underline{n} \cdot c \nabla u$ in (1b) represents the heat flow into the domain and is calculated internally and the term λ in (1b) is zero because mixed boundary conditions are not applied

in this example. Fig. 2 shows the complete FemLab code. The default values of all PDE and boundary coefficients are 0. Also some comments (%) are included for better

```
\partial \theta
    = \nabla \cdot (D(\theta) \cdot \nabla \theta)
                        on 0 < x < 0.024,
 ∂t
\theta = 0.27
                         on x = 0,
 \partial \theta
    = 0
                         on x = 0.024
\partial x
fem.geom=solid1([0 0.024]);
                                       % 1D geometry
                                          diffusivity fired clay brick, type I
Dw=['(3.2e-9)*exp(29*u)'];
                                       8
fem.dim=1;
                                       °
                                          one component
                                          c equals diffusivity
fem.equ.c={Dw};
                                       °
fem.equ.da={1}
                                       %
                                          da equals 1
                     0\};
fem.bnd.h={1
                                       °
                                          boundary value
fem.bnd.r={0.27 0};
                                       ÷
                                         boundary value
```

Fig. 5. The PDE and boundary conditions (upper) and the corresponding part of the FemLab code (lower) of the 1D moisture transport problem.



Fig. 6. Measured and simulated moisture content profiles versus lambda of the 1D moisture transport problem. Left-hand side: measured moisture contents, right-hand side: simulated moisture contents.

understanding of the code. The initial mesh is presented in Fig. 3, and the solution in Fig. 4. This example shows the transparency, easy-to-use and flexibility of PDE models in FemLab.

3. Testcases for reliability

In [1] examples are already present which show the accuracy and reliability of FemLab. However, there are no validations for typical building physics problems. Specific for validating building physics simulations in FemLab, this section deals with two (very different) time dependent non-linear problems. Each problem is solved and compared with measured data.

3.1. 1D moisture transport in a porous material

The water absorption in an initially dry brick cylinder (length 24 mm) is studied. All sides except the bottom are sealed. This side is submerged in water at t = 0 s. The



Fig. 7. The geometry and boundary conditions for the 2D-airflow problem.

```
U-momentum equation
    \frac{\partial u}{\partial t} = -\frac{\partial (uu)}{\partial r} - \frac{\partial (vu)}{\partial v} - \frac{\partial p}{\partial t} + \frac{1}{\text{Re}} \nabla^2 u
V-momentum equation
             \frac{\partial v}{\partial v} = -\frac{\partial(uv)}{\partial v} - \frac{\partial(vv)}{\partial v}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    \frac{\partial p}{\partial y} + \frac{1}{\operatorname{Re}} \nabla^2 v + \frac{Gr}{\operatorname{Re}^2} T
         \partial t \qquad \partial x
                                                                                                                                                                                                                                                                                                                                       ∂y
Continuty equation
    \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
Energy equation
             \frac{\partial T}{\partial t} = -\frac{\partial(uT)}{\partial t}
                                                                                                                                                                                                                                                                                                                                                          \frac{\partial(vT)}{\partial(vT)} + \frac{1}{\partial(vT)}
             ∂t
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              Re Pr
                                                                                                                                                                                                      \partial x
                                                                                                                                                                                                                                                                                                                                                                                               ∂v
```

```
%Variables: u1=u; u2=v; u3=p; u4=T
                      ulx = dul/dx, etc.
         °
eta=1/Re;beta=Gr/(Re*Re);alpha=1/(Re*Pr); %parameters
fem.dim=4;
fem.equ.da={{1; 1; 0; 1}};
fem.equ.F={{ '-(u1.*ulx+u2.*uly+u3x)';...
             '-(u1.*u2x+u2.*u2y+u3y)+beta*u4';...
            '-(u1x+u2y)';...
            '-(u1.*u4x+u2.*u4y)'};
fem.equ.ga={{{'-eta*ulx'; '-eta*uly'};...
{'-eta*u2x'; '-eta*u2y'};...
              0;...
              {'-alpha*u4x'; '-alpha*u4y'}};
fem.bnd.r={ {'-u1'; '-u2'; 0; '0-u4'} ...
              '-u1'; '-u2'; 0; '0-u4'
             {'1-u1'; '-u2'; 0; '1-u4'} ...
              '-u1'; '-u2'; 0; '0-u4'} ...
              '-u1'; '-u2'; 0; '0-u4'} ...
              '-ul'; '-u2'; 0; '0-u4'} ...
             [0; 0; '-u3'; 0} ...
             {'-u1'; '-u2'; 0; '0-u4'} };
fem=femdiff(fem); % calculate divergence gamma on domain
```

Fig. 8. The PDE model and the corresponding FemLab code for the 2D-airflow problem.

PDE and boundary conditions for this problem are shown in Fig. 5 (upper part). The corresponding FemLab code for defining the geometry, PDE and boundaries is shown in the lower part of Fig. 5. The coefficient form (1) is used. The results of measured water absorption of several brick materials based on [3] are shown in Fig. 6, left-hand side. The moisture profiles are shown versus *lambda* (*lambda* is defined by the position divided by the square root of the time). For each material the diffusivity as a function of the moisture content is given [3] and used to simulate the corresponding profiles. The simulation results in Fig. 6, right-hand side, show a good agreement with the measured profiles.

3.2. 2D airflow in a room

This example from [4] deals with the velocity and temperature distribution in a room heated by a warm air stream. In Fig. 7 the geometry and boundary conditions are presented. The problem is modeled by 2D incompressible flow using the Boussinesq approximation with constant properties for the Reynolds and Grasshof numbers. The general form (2) is used for this type of non-linear problem. In Fig. 8, the PDE model, the corresponding PDE coefficients of (2a) and the corresponding part of the code are given. In [4] the problem is solved and validated with measurements for several combinations of Re and Gr. In Fig. 9 these results are presented. The left-hand side shows the results obtained by [4] and the right side show the corresponding FemLab results. The results are in good agreement.

3.3. Discussion on reliability

The test cases in this section show that FemLab is very reliable even for a highly non-linear problem such as



Fig. 9. Dimensionless temperature contours comparison of the validated simulation results of [4] (left-hand side) with the FemLab results (right-hand side) for the 2D-airflow problem.

convective airflow. Furthermore, for all the simulation results presented in this paper, the default mesh generation and solver are used. So good results can be obtained without a deep understanding of the gridding and solving techniques. The simulation times (based on processor Pentium 3, 500 MHz) ranged from fast (order \sim seconds) for linear problems such as the 2D thermal bridge example, medium (order \sim minutes) for non-linear problems in the coefficient form such as the 1D moisture transport example to high (order \sim hours) for highly non-linear problems in the general form such as the airflow problem.

4. 3D combined heat and moisture transport

In this section the challenging problem of simulating combined heat and moisture transport for a 3D geometry is presented. The problem is based on a 2D-problem [5] but is now extended to a 3D problem. A brick specimen, initially dry and two of the flanks sealed, is placed with its lower surface about 1 cm under water. In Fig. 10, the geometry is presented. The PDE model, boundary conditions and corresponding part of the code written in the coefficient form (1), are shown in Fig. 11. In the PDE model 3 material



Fig. 10. Geometry for the 3D combined heat and moisture transport.

$\frac{\partial w}{\partial t} = \nabla \cdot (Dw(w) \nabla w + DF(T, w) \nabla T)$ $\frac{\partial T}{\partial t} = \nabla \cdot (DT(w) \nabla T)$ $w(t = 0) = w0, T(t = 0) = T0,$ sealed sides : $\nabla T = \nabla w = 0$ open sides : $\nabla T = A(T - Ti), \nabla w = B(w - wi)$ bottom : $w(z = 0) = w \max , T(z = 0) = T0$								
<pre>fem.dim=2; fem.form='coefficient fem.equ.da={{1; 1}}; fem.equ.c={'Dw(w)' 'I</pre>	_'; DF(T,w)' 0 'DT(w)'};							
fem.bnd.g={ {0 0 fem.bnd.q={ {0 0 fem.bnd.h={ {1 5 fem.bnd.r={ {wmax 5	D} {B*wi A*Ti }{C D} {B A }{C 1} {0 0}{C TO} {0 0}{C 0}{0}{C}	0){0 0}{0 0){0 0}{0 0){0 0}{0 0}{0}{0}{0}{0}{0}{0}{0}{0}{0}{0}{0}{0}{0	0}{0 0} 0}{0 0} 0}{0 0} 0}{0 0} 0}{0 0}					

Fig. 11. The PDE model and the corresponding FemLab code for the 3D combined heat and moisture transport.

properties functions are defined: $D_w(w)$, $D_F(T,w)$ and $D_T(w)$. These functions are calculated using the material properties presented in [5] and are shown in Fig. 12. The results of the simulated moisture distribution initially and after 5, 10 and 70 days are visualized by slices and planes with equivalent moisture content, in Fig. 13. The extension of 3D modeling with different materials is left over for future research.

5. Conclusions

FemLab is evaluated as a solver for building physics problems based on partial differential equations (PDEs).



Fig. 12. The material properties for the PDE coefficients $D_w(w)$, $D_T(w)$ and $D_F(T, w)$ based on [5].

Typical building physics problems such as moisture transport in a porous material, dynamic airflow and combined heat and moisture transport are relatively easy to model. The simulation results of the models used including 1D–3D geometries show a good agreement with measurements. The FemLab software is written in the MatLab environment and therefore it is possible to make use of the visualisation tools, toolboxes and all other programs written in MatLab. The evaluation illustrates the powerful and flexible nature of FemLab for solving scientific and engineering building physics problems.



Fig. 13. 3D moisture distribution profiles for 0 days (top), 5 days (middle) and 70 days (bottom). Left-hand side shows vertical slices and right-hand side shows iso surfaces.

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